LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2013**

MT 4812 - PARALLEL INTERCONNECTIONS NETWORKS

Date : 30/04/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

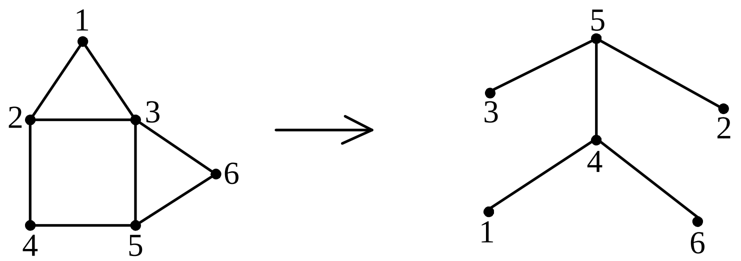
ANSWER ALL QUESTIONS

I (a) Define interconnection network and show how it may be modeled by a simple graph.

[OR]

(b) What type of graphs is used to model cross-bar switches? Define a strongly connected graph with an example. (5)

(c) Define an embedding of a graph G into H. Explain the parameters (i) dilation, (ii) congestion, (iii) dilation-sum, and (iv) congestion-sum of an embedding. For embedding f show below determine each of the parameters. (15)



[OR]

(d) (i) Let G be a connected undirected graph with order $(n \geq 3)$ and the maximum degree $d \geq 2$.

Then prove that $d(G) \geq \begin{cases} \left\lfloor \frac{1}{2} n \right\rfloor & for\ d = 2 \\ \left\lceil log_{(d-1)} \frac{n(d-2)+2}{d} \right\rceil & for\ d \geq 3 \end{cases}$

(ii) Discuss the role of planar graphs in the layout of VLSI circuits. (10 + 5)

II (a) Let G be a graph of order n. Then prove that for any $\theta \in Aut(G)$, its restricition to X is an isomorphism between G[X] and G[$\theta(X)$] for any non-empty $X \subseteq V(G)$ where $\theta(X) = \{y \in V(G): y = \theta(x), x \in X\}$

[OR]

(b) Prove that the converse of $\overleftarrow{C_\Gamma(S)}$ of a cayley graph $C_\Gamma(S)$ is also a cayley graph. Also list 3 properties of a cayley graph. (5)

(c) Define a line graph of an undirected graph. Let G be a simple undirected graph and L(G) be the line graph of G. Prove the following:

(i) L(G) is simple and $v\big(L(G)\big) = \varepsilon(G)$.

(ii) $d_{L(G)}(e) = d_G(x) + d_G(y) - 2$ for any $e = xy \in E(G)$, and hence $\delta(L(G) = \xi(G)$. In particular, L(G) is (2d – 2) –regular if G is d-regular.

(iii) For any $x \in V(G)$, the subgraph of L(G) induced by the edges incident with $x \in V(G)$ is a complete graph.

(iv) $\varepsilon\big(L(G)\big) = \frac{1}{2}\sum_{x \in V(G)}(d_G(x))^2 - \varepsilon(G)$.

(v) For a connected undirected graph G, $L(G) \cong G$ if and only if G is an undirected cycle.

[OR]

(d) Define cayley graph. Prove that a cayley graph is regular and does not have a loop. Generate the cayley graph when $G = \{0, 1, 2, 3, 4, 5, 6,\}$ is the additive group of modulo 7 and $s = \{1, 2, 4\}$.

(15)

III (a) Define Hypercubes. Also draw Q_4

[OR]

(b) Write 5-bit Gray code G_5. (5)

(c) (i) For any given vertex x of Q_n ,prove that there exists a unique vertex y such that the distance $d(Q_n; x, y) = n$. Also prove that there are n internally disjoint (x, y)- paths of length n.

(ii) Let x and y be two vertices in Q_n and $d(Q_n; x, y) = d$. Then prove that there exist a d-dimensional subcube in Q_n in which there are d internally disjoint (x, y)- paths of length d. Also prove that there exist n internally disjoint (x, y)- paths of length d in Q_n such that d of which are of length d, otherwise of length $d+2$.

(7 + 8)

[OR]

(d) (i) Prove that $\nu(G_1 \times G_2) = \nu(G_1)\nu(G_2)$ and $\varepsilon(G_1 \times G_2) = \nu(G_1)\varepsilon(G_2) + \nu(G_2)\varepsilon(G_1)$.

(ii) Prove that $2T_{n-1}$ can be embedded into Q_{n+1} with dilation 1. (8 + 7)

IV (a) Draw $B(2,3)$ and construct an euler circuit in it.

[OR]

(b) Define circulant networks and draw $G(8; \pm\{1,2,3\})$ (5)

(c) (i) Write the procedure to construct CCC(n) from WBF(n). Also construct CCC(3) from WBF(3)

(ii) Let ρ_m be a minimum routing in a wheel W_7. Find $\pi(W_7, \rho_m)$. (9 + 6)

[OR]

(d) Define De Bruijin networks using d-ary sequence, line graphs and arithmetic method. Prove that these three definitions are equivalent. (15)

V (a) Write a note on forwarding index of routing.

[OR]

(b) Prove that $\tau(Q_n) = 2^{n-1}(n-2)+1$. (5)

(c) Let G be a strongly connected digraph n, prove that

$$\frac{1}{n}\sum_{y \in V}\sum_{x(\neq y) \in V}\left(d(G; x, y) - 1\right) \leq \tau(G) \leq (n-1)(n-2)$$

Also prove that the upper bound can be attained and, the lower bound of $\tau(G)$ can be attained if and only if there exists a minimum routing ρ_m in G for which the load of all vertices is the same.

[OR]

(d) Prove that $P(n,2)=\left\lceil \frac{n}{3} \right\rceil$ if $n \geq 4$. (15)

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